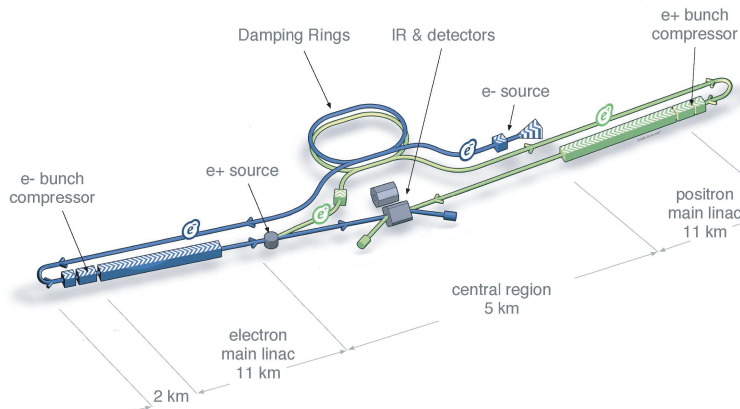
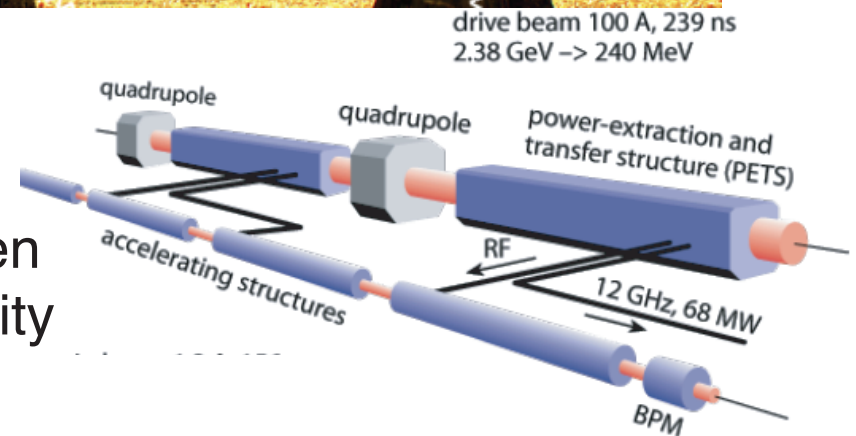


# Linear Collider (not just ILC) Sensitivity Studies

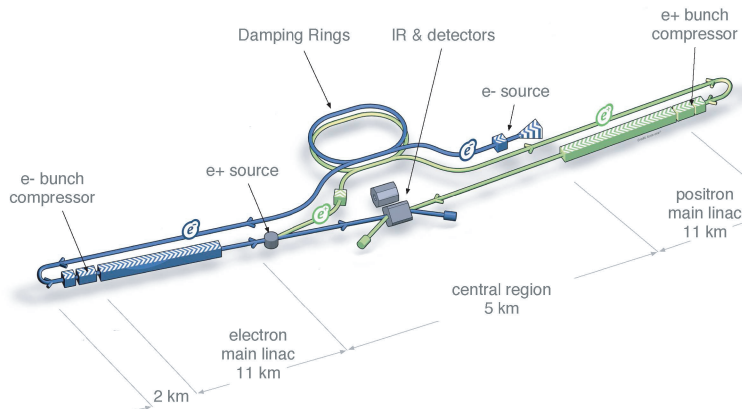


Rick Van Kooten  
Indiana University

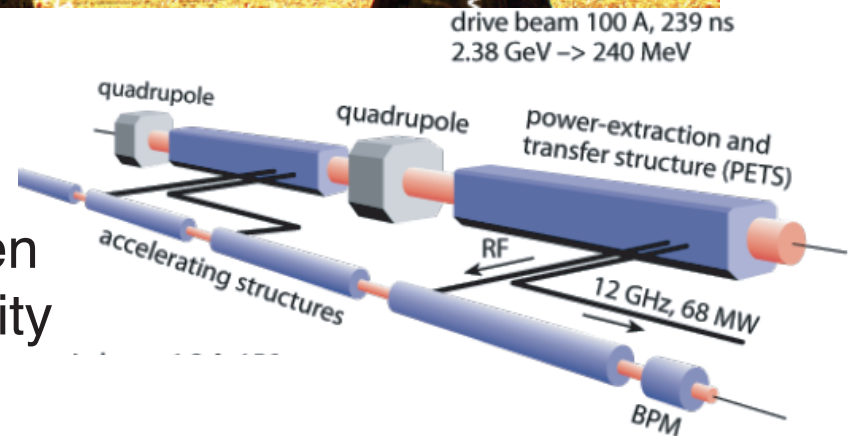


drive beam 100 A, 239 ns  
2.38 GeV  $\rightarrow$  240 MeV

# $e^+e^-$ (not just ILC) Sensitivity Studies



Rick Van Kooten  
Indiana University



## Outline

- (New) baselines for linear collider options
- Detectors
- $W$ -pair production:  $e^+e^- \rightarrow W^+W^-$   
Anomalous triple gauge couplings (aTGC)
- Triboson production:  $e^+e^- \rightarrow VVV$   
Anomalous quartic gauge couplings (aQGC)
- Vector boson scattering:  $e^+e^- \rightarrow \nu_e \bar{\nu}_e W^+W^-$   
Anomalous quartic gauge couplings (aQGC)
- Constraints from global Higgs fits
- Summary

# Acknowledgements

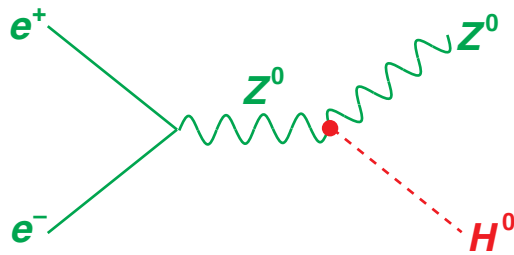
## Reporting on the work of many others

- Snowmass EW Report, [arXiv:1310.6708v1 \[hep-ph\]](#)
- ILC Technical Design Report | Vol. 2: Physics, [arXiv:1306.6352 \[hep-ph\]](#)
- Exploring Quantum Physics at an ILC, [arXiv:1307.3962 \[hep-ph\]](#)
- CLIC Snowmass Report, [arXiv:1307.5288v3 \[hep-ex\]](#)
- CLIC Physics & Detectors: CDR, [arXiv:1202.5940 \[physics.ins-det\]](#)
- First Look at the Physics Case of TLEP, [arXiv:1308.6176 \[hep-ex\]](#)
- Physics Interplay of the LHC and the ILC, [hep-ph/0410364](#)
- Determination of New Electroweak Parameters  
at the ILC — Sensitivity to New Physics, [arXiv:hep-ph/0604048v1](#)
- Study of Charged Current Triple Gauge Couplings  
at TESLA, [LC-PHSM-2001-022](#)
- Constraining anomalous Higgs interactions, [arXiv:1207.1344v3 \[hep-ph\]](#)

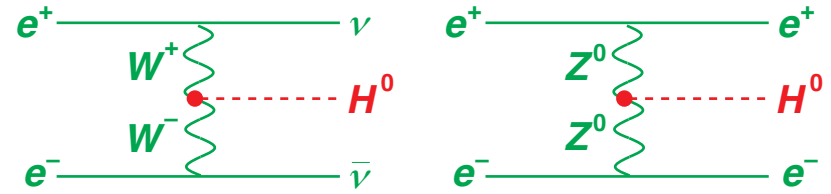
...and references therein



# ILC Base Program



- "Higgs"strahlung



- Fusion

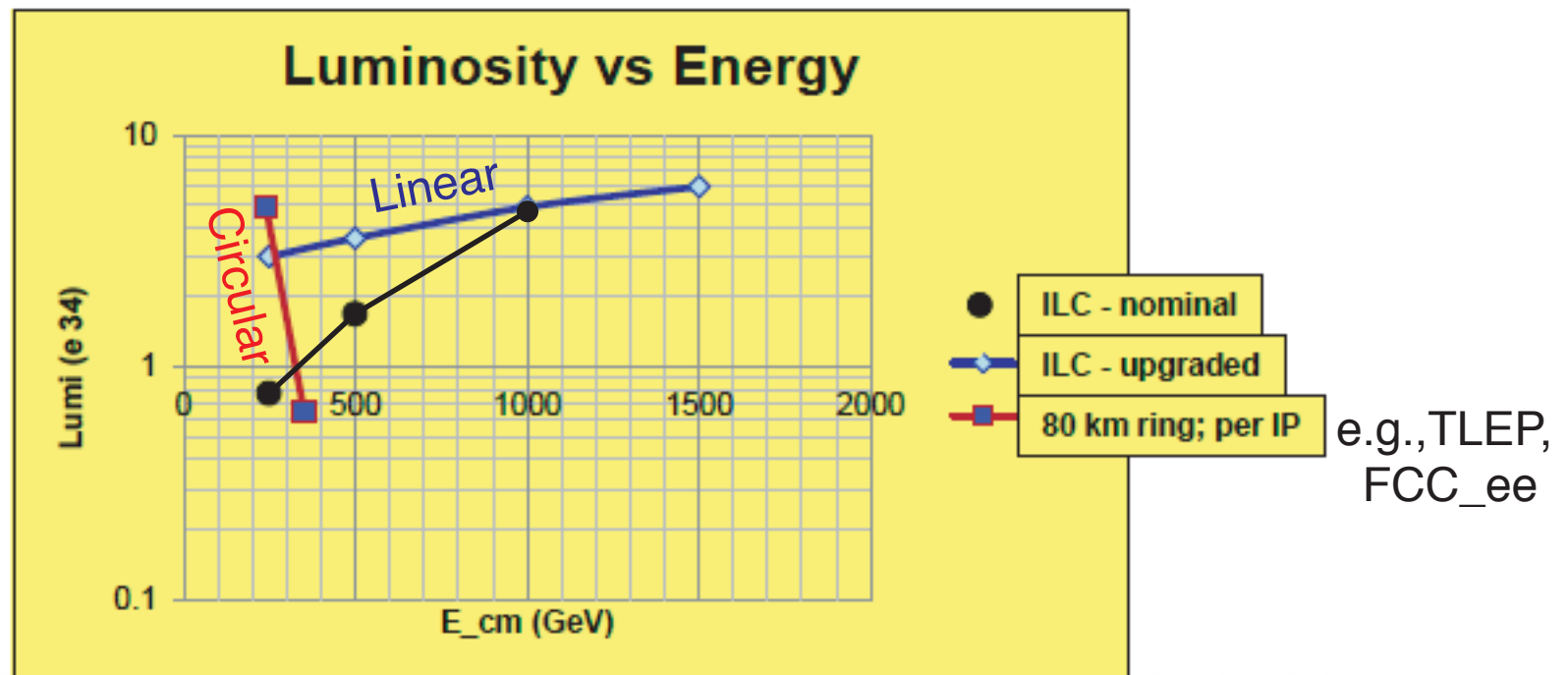
Typical ILC program, 3 – 5 years each energy:

	250 GeV	350 GeV	500 GeV	1 TeV	1.5 TeV	3 TeV
$\sigma(e^+e^- \rightarrow ZH)$	300 fb	129 fb	57 fb	13 fb	6 fb	1 fb
$\sigma(e^+e^- \rightarrow \nu\bar{\nu}H)$	18 fb	30 fb	75 fb	210 fb	309 fb	484 fb
Int. Luminosity	250 fb <sup>-1</sup>	350 fb <sup>-1</sup>	500 fb <sup>-1</sup>	1 ab <sup>-1</sup>	1.5 ab <sup>-1</sup>	2 ab <sup>-1</sup>
# ZH events	75,000	45,500	28,500	13,000	7,500	2,000
# $\nu\bar{\nu}H$ events	4,500	10,500	37,500	210,000	460,000	970,000

Polarized

# ILC Luminosity Upgrade

- ILC base program frozen long ago for global design effort (GDE) and technical design report → necessarily conservative



- With minimal cost impacts, possible luminosity upgrade also considered for Snowmass studies:

$$\mathcal{L} = 0.75 \rightarrow 3.0 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1} \quad \int \mathcal{L} dt \begin{matrix} + \text{optimistic} \\ - \text{pessimistic} \end{matrix}$$

## Benchmark Programs

...considered for Snowmass studies

*pp* machines:

	LHC	HL-LHC	HE-LHC	VLHC
$\sqrt{s}$ (TeV)	14	14	33	100
$\int \mathcal{L} dt$ (fb <sup>-1</sup> )	300	3000	3000	3000

Linear  $e^+e^-$  machines:

*Luminosity  
Upgrade*

	ILC500	ILC1000	ILC1000-up	CLIC
$\sqrt{s}$ (TeV)	250/500	250/500/1000	250/500/1000	350/1400/3000
$\int \mathcal{L} dt$ (fb <sup>-1</sup> )	250+500	250+500+1000	1150+1600+2500	500+1500+2000

Run scenarios:

*Considered for  
Higgs projections*

Facility	HL-LHC	ILC	ILC(LumiUp)	CLIC
$\sqrt{s}$ (GeV)	14,000	250/500/1000	250/500/1000	350/1400/3000
$\int \mathcal{L} dt$ (fb <sup>-1</sup> )	3000/expt	250+500+1000	1150+1600+2500	500+1500+2000
$\int dt$ (10 <sup>7</sup> s)	6	3+3+3	(ILC 3+3+3) + 3+3+3	3.1+4+3.3

# Benchmark Programs

...considered for Snowmass studies

*pp* machines:

	LHC	HL-LHC	HE-LHC	VLHC
$\sqrt{s}$ (TeV)	14	14	33	100
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Linear  $e^+e^-$  machines:

	ILC500	ILC1000	ILC1000-up	CLIC
$\sqrt{s}$ (TeV)	250/500	250/500/1000	250/500/1000	350/1400/3000
$\int \mathcal{L} dt$ (fb <sup>-1</sup> )	250+500	250+500+1000	1150+1600+2500	500+1500+2000

*Luminosity  
Upgrade*

Run scenarios:

*Results shown  
here baseline  
program*

*Considered for  
Higgs projections*

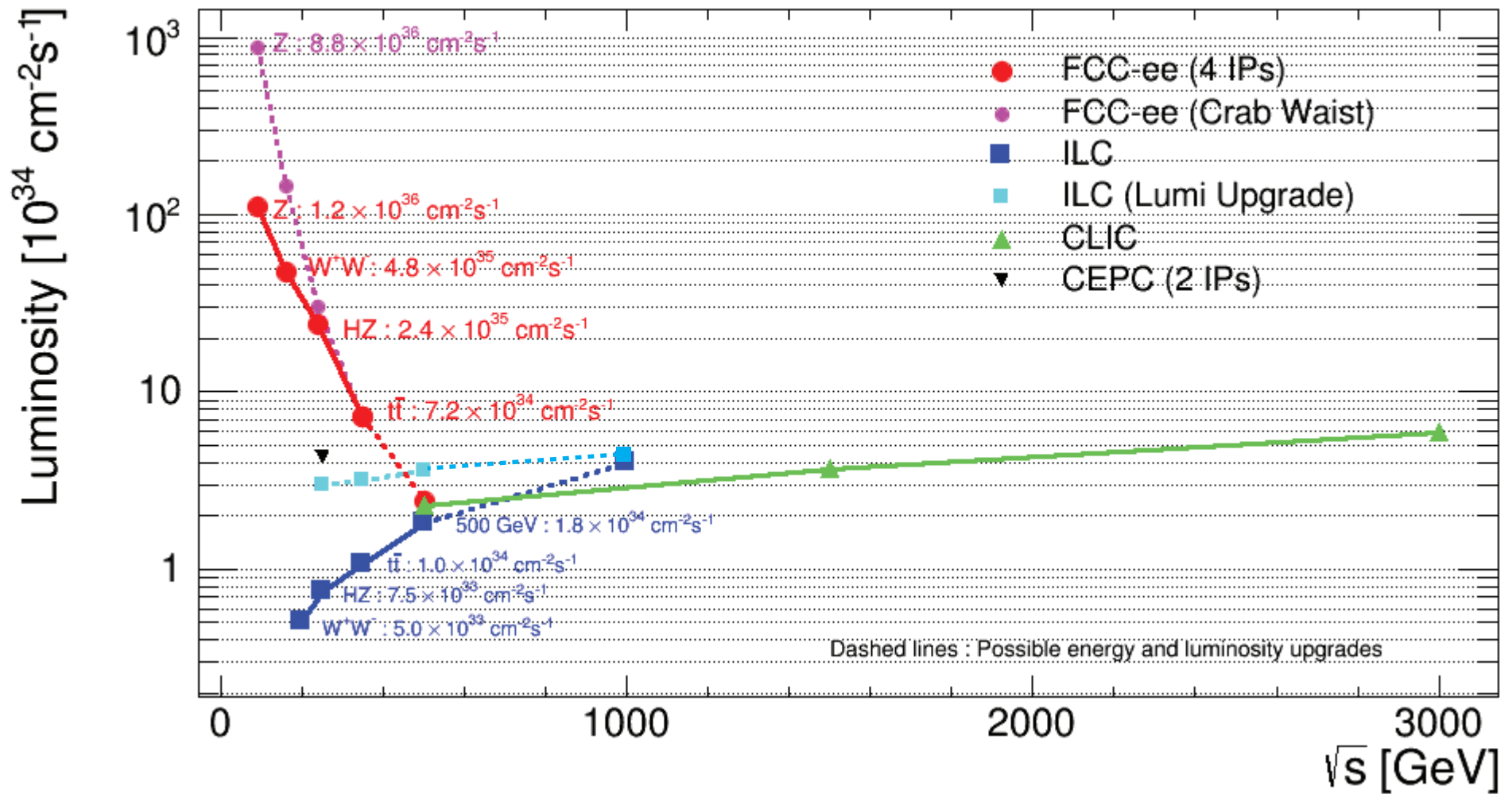
Facility	HL-LHC	ILC	ILC(LumiUp)	CLIC
$\sqrt{s}$ (GeV)	14,000	250/500/1000	250/500/1000	350/1400/3000
$\int \mathcal{L} dt$ (fb <sup>-1</sup> )	3000/expt	250+500+1000	1150+1600+2500	500+1500+2000
$\int dt$ (10 <sup>7</sup> s)	6	3+3+3	(ILC 3+3+3) + 3+3+3	3.1+4+3.3

...so results *may* be a bit conservative



# Benchmark Programs

...considered for Snowmass studies

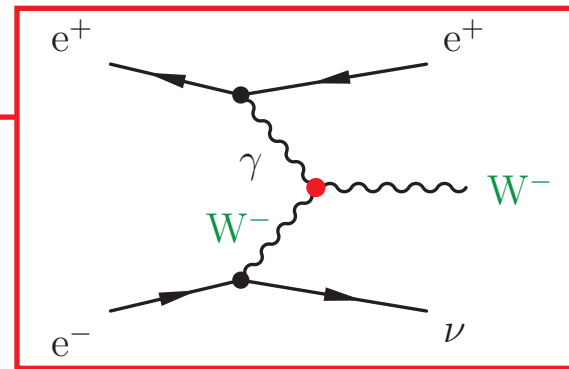
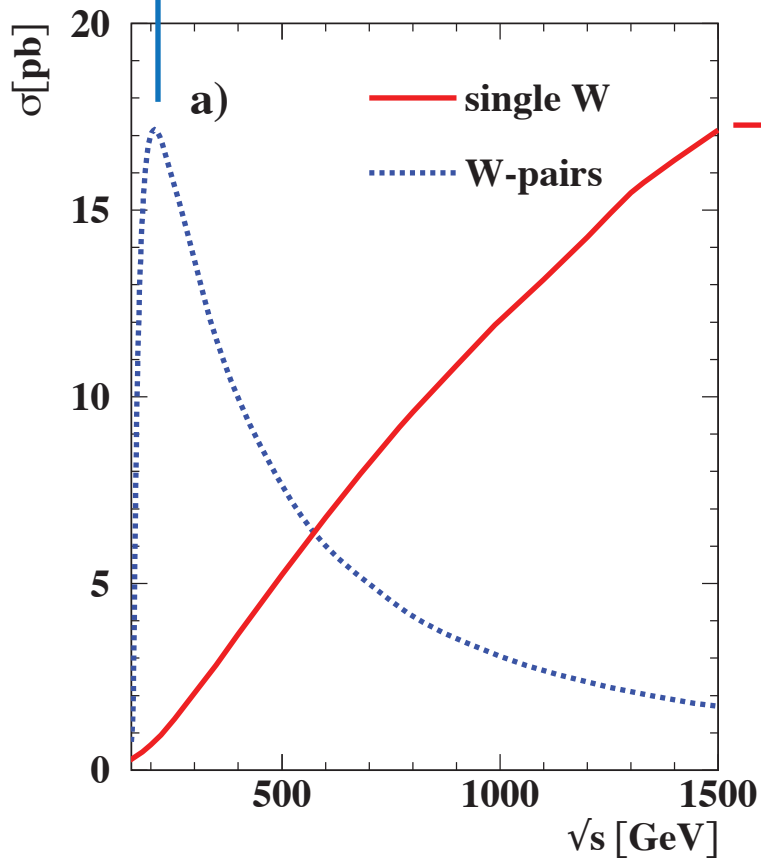
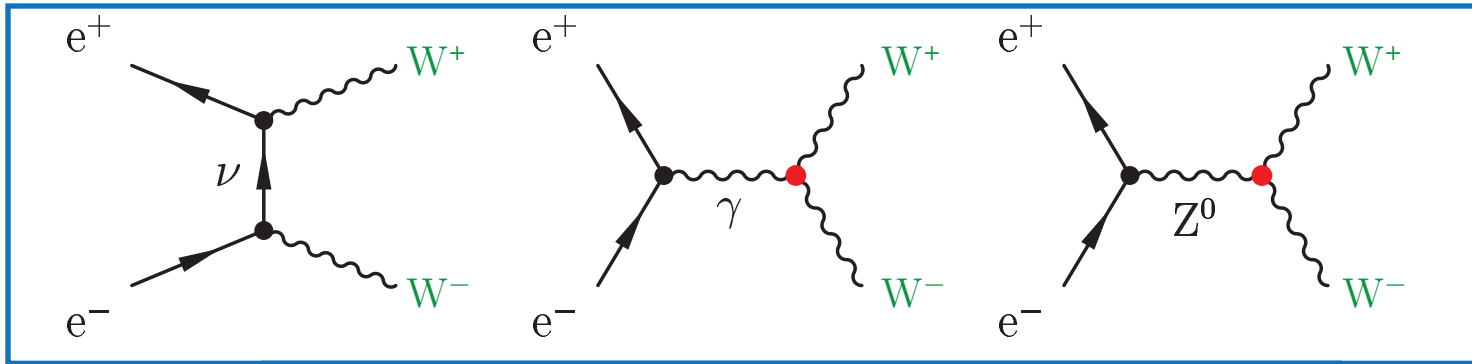


## Advantages of $e^+e^-$

- absence of parton distribution functions
  - known center of mass
- “democratic” in production of signal & background
  - far smaller QCD background
  - more electroweak, smaller theoretical uncertainties
- cleanliness of final state (modulo, e.g., earlier comments regarding NLO corrections EW VBS)
  - no beam remnants
- beam structure, msec between bunch trains
  - essentially triggerless
  - advanced detector hardware & excellent resolutions
- longitudinal polarization of beams, ( $V-A$ ) nature of  $W/Z$ 
  - $\mathcal{P}(e^-) = 80 - 90\%$ ,  $\mathcal{P}(e^+) = 30 - 60\%$
  - $e_L$  and  $e_R$  different multiplets, access completely different couplings

# Making $W$ 's

...dominant  $W$  production processes at a linear collider



- This is the vector boson fusion diagram – not studied much for TGC's since  $W$ -pair process so powerful

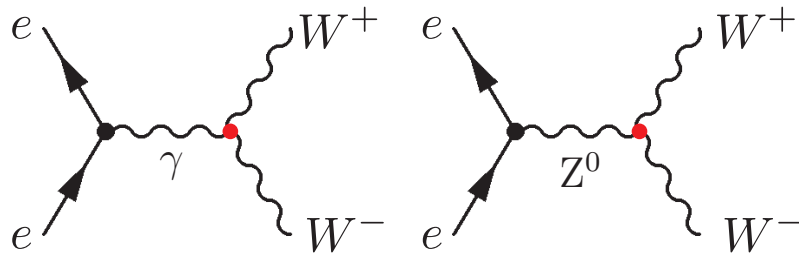
$$e^+e^- \rightarrow W^+W^-$$

...for triple gauge couplings

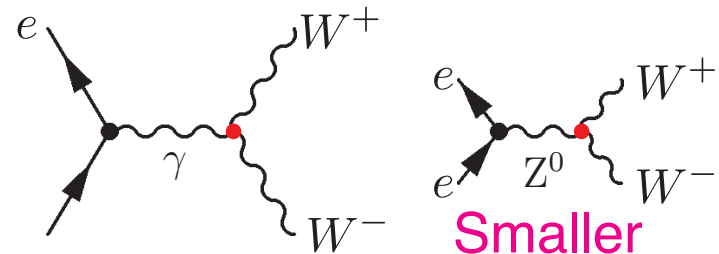
Production at linear colliders would be first time  $W^+W^-$  production measured with polarized beams

$$\sigma = \frac{1}{4}(1 - \mathcal{P}^+)(1 + \mathcal{P}^-)\sigma_R + \frac{1}{4}(1 + \mathcal{P}^+)(1 - \mathcal{P}^-)\sigma_L$$

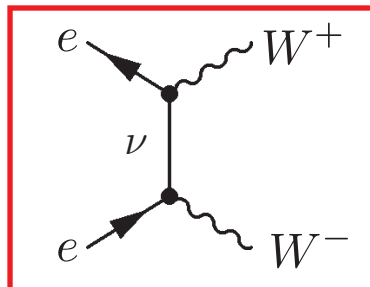
Left-handed electrons



Right-handed electrons



Disentangle the  $WW\gamma$  and  $WWZ$  couplings



Turn off

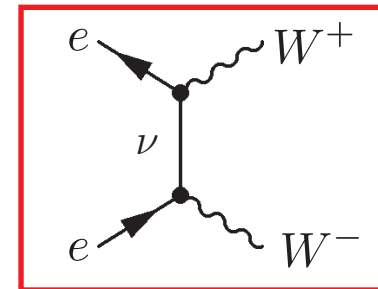
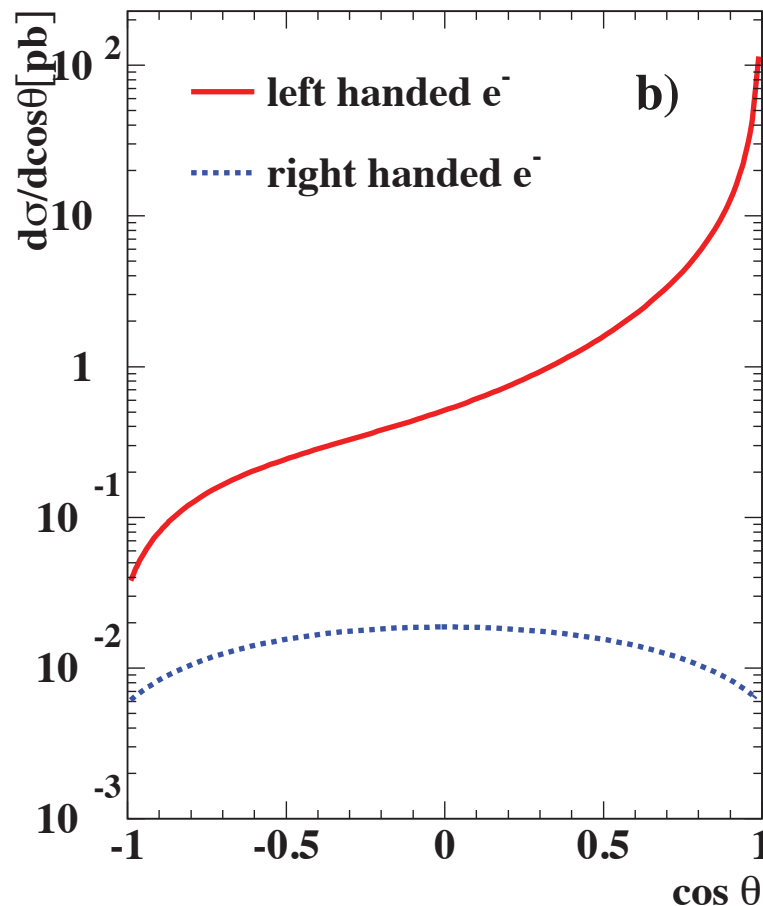


$$e^+e^- \rightarrow W^+W^-$$

...for triple gauge couplings

Production at linear colliders would be first time  $W^+W^-$  production measured with polarized beams

$$\sigma = \frac{1}{4}(1 - \mathcal{P}^+)(1 + \mathcal{P}^-)\sigma_R + \frac{1}{4}(1 + \mathcal{P}^+)(1 - \mathcal{P}^-)\sigma_L$$

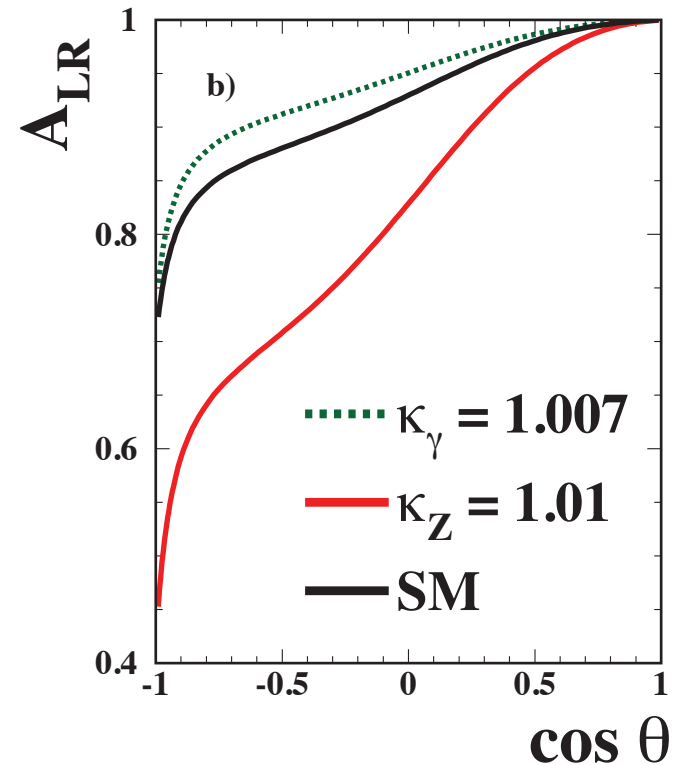
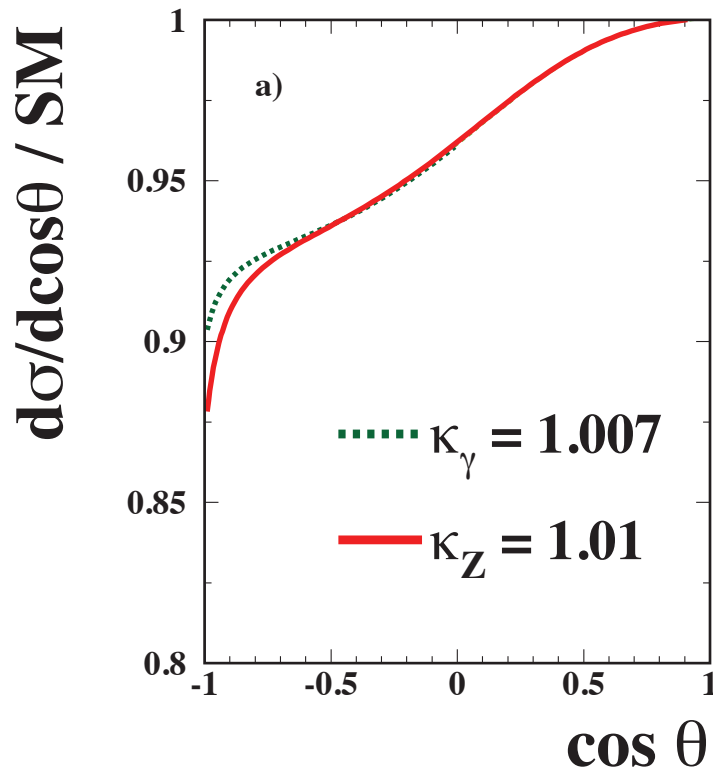


$$e^+e^- \rightarrow W^+W^-$$

...for triple gauge couplings

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$$\sigma = \frac{1}{4}(1 - \mathcal{P}^+)(1 + \mathcal{P}^-)\sigma_R + \frac{1}{4}(1 + \mathcal{P}^+)(1 - \mathcal{P}^-)\sigma_L$$



- Polarization increases sensitivity to aTGC's

$$e^+e^- \rightarrow W^+W^-$$

...for triple gauge couplings

- Usual multidimensional fits to  $W$ -production angles and angles of  $W$ -decay products, different polarizations

- $W$ 's boosted, better resolution on  $W$ -production angle than LEP2

- Use all three decay topologies:

$$W \rightarrow qq', W \rightarrow qq'$$

$$W \rightarrow qq', W \rightarrow \ell\nu$$

$$W \rightarrow \ell\nu, W \rightarrow \ell\nu$$

Most sensitive  
select with good efficiency,  
rather low background

$\pm 1\sigma$  uncertainties to  
per mille level or better

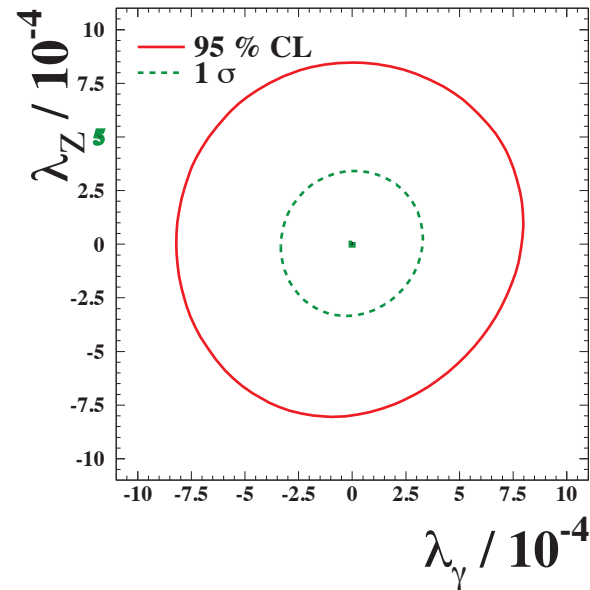
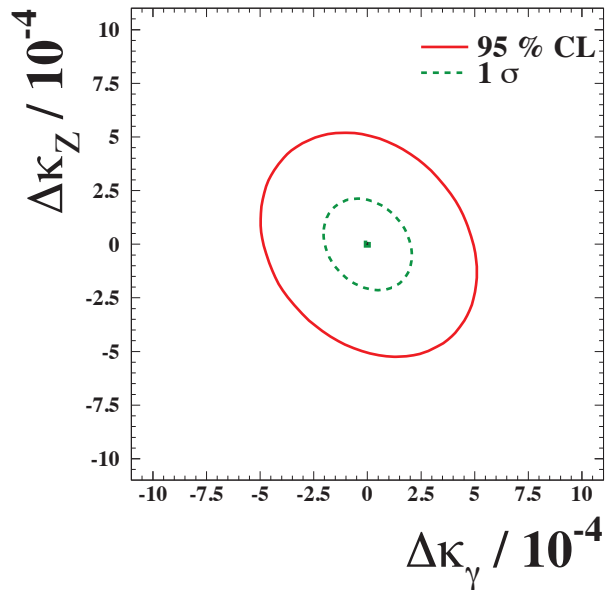
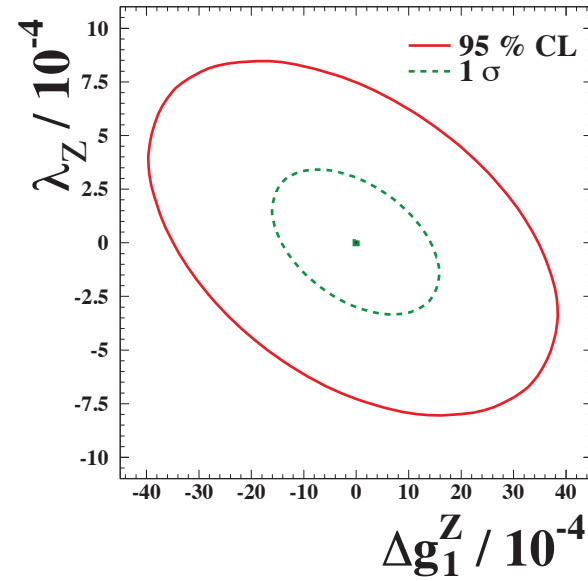
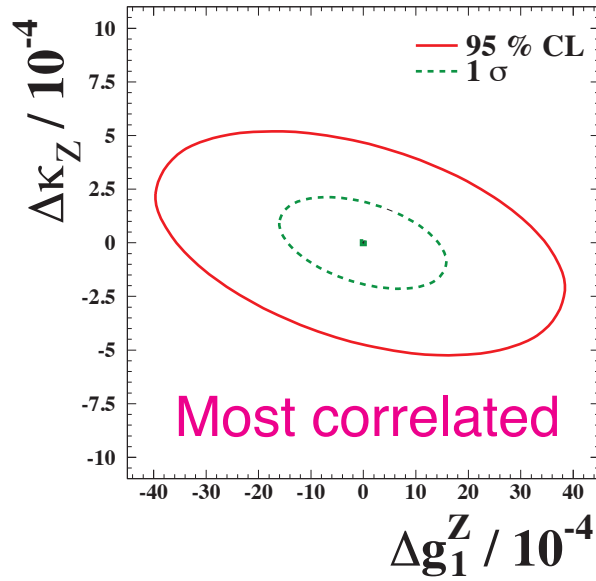
coupling	error $\times 10^{-4}$	
	$\sqrt{s} = 500 \text{ GeV}$	$\sqrt{s} = 800 \text{ GeV}$
C,P-conserving, $SU(2) \times U(1)$ relations:		
$\Delta g_1^Z$	2.8	1.8
$\Delta \kappa_\gamma$	3.1	1.9
$\lambda_\gamma$	4.3	2.6
C,P-conserving, no relations:		
$\Delta g_1^Z$	15.5	12.6
$\Delta \kappa_\gamma$	3.3	1.9
$\lambda_\gamma$	5.9	3.3
$\Delta \kappa_Z$	3.2	1.9
$\lambda_Z$	6.7	3.0
not C or P conserving:		
$g_5^Z$	16.5	14.4
$g_4^Z$	45.9	18.3
$\tilde{\kappa}_Z$	39.0	14.3
$\tilde{\lambda}_Z$	7.5	3.0

$$\mathcal{L} = 500 \text{ fb}^{-1} \quad 1000 \text{ fb}^{-1}$$

$$\mathcal{P}^- = 80\%, \mathcal{P}^+ = 60\%$$

$$e^+e^- \rightarrow W^+W^-$$

...for triple gauge couplings

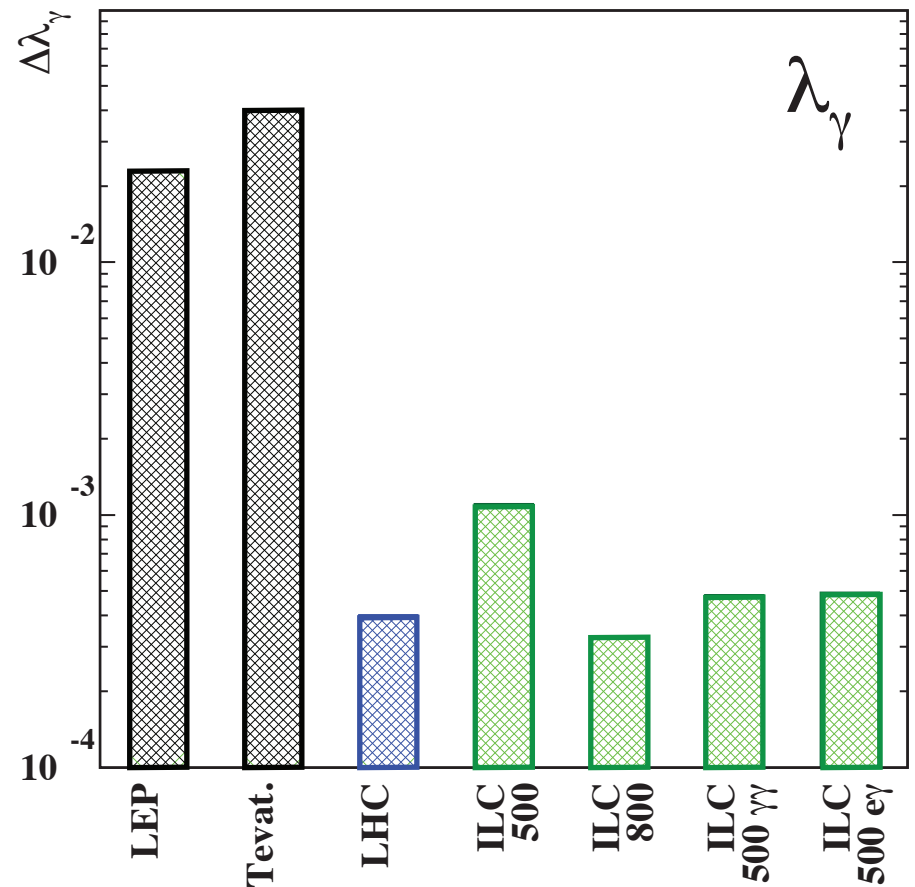
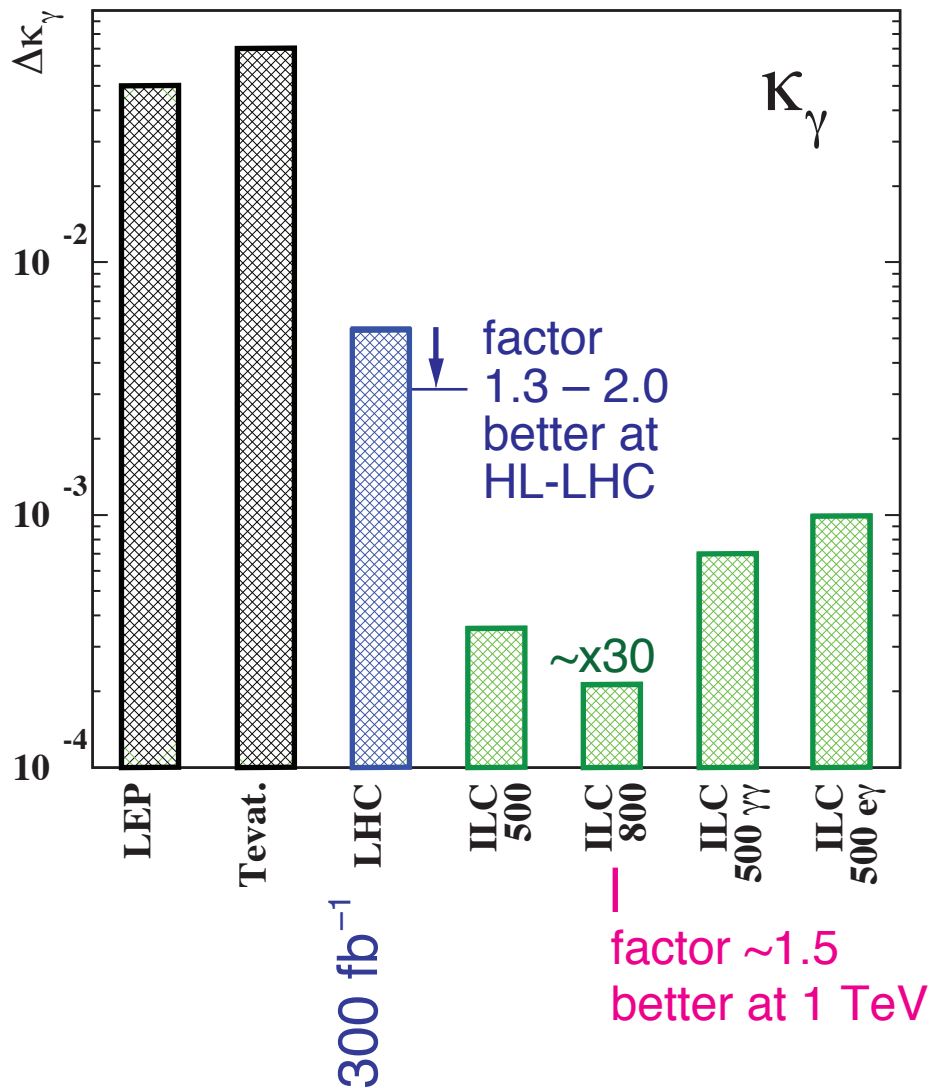




# aTGC's in Context

...from  $e^+e^- \rightarrow W^+W^-$

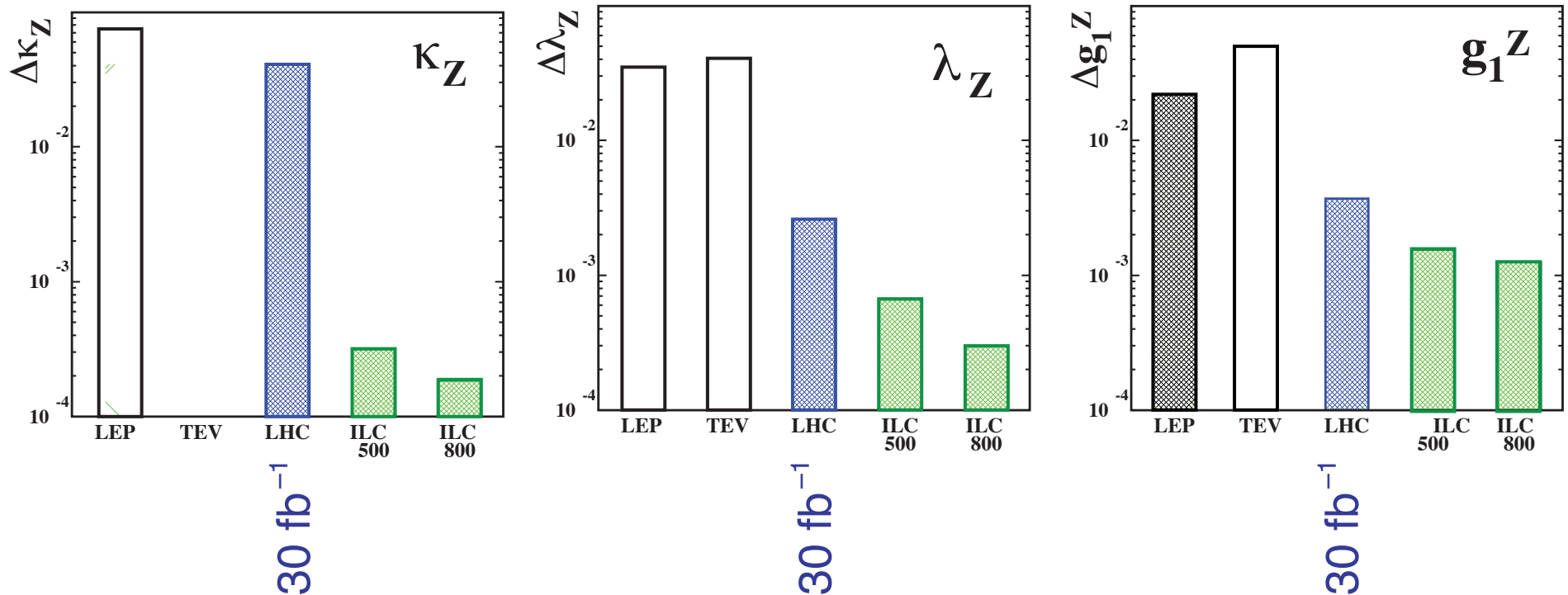
- large improvement in precision in many TGC's



## aTGC's in Context

...from  $e^+e^- \rightarrow W^+W^-$

- all precisions improve, many by large factors



From hep-ph/0410364, updated

## TLEP?

## FCC-ee: Future Circular ee Collider

- At lower energies, TLEP would have insane luminosities
- ILC at Z peak has “GigaZ” program  
TLEP at Z peak has “TeraZ” program
- $2 \times 10^8$   $W^+W^-$  pairs at threshold ( $\sim 1/10$ ) and above
- “...measurements to be performed by TLEP at this centre-of-mass energy need to be thoroughly reviewed by the starting design study”

## Triple VB Production

...for quartic gauge couplings

$$\sigma(e^+e^- \rightarrow VVV) \propto \frac{1}{s}$$

Limits usefulness to subprocess energies in the lower range where cross section of fusion process still small

$$\sigma_{\text{VBS}}(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-) \propto \log(s)$$

$$\begin{array}{l} e^+e^- \rightarrow ZZZ \\ \rightarrow WWZ \\ \rightarrow WW\gamma \end{array} \left. \begin{array}{l} \textcolor{violet}{ZH} \\ \textcolor{violet}{\hookrightarrow WW} \\ \textcolor{violet}{\hookrightarrow ZZ} \end{array} \right\} \begin{array}{l} \text{Present in spectrum} \\ \text{Complementary (and present at lower energies)} \end{array}$$



# Triple VB Production

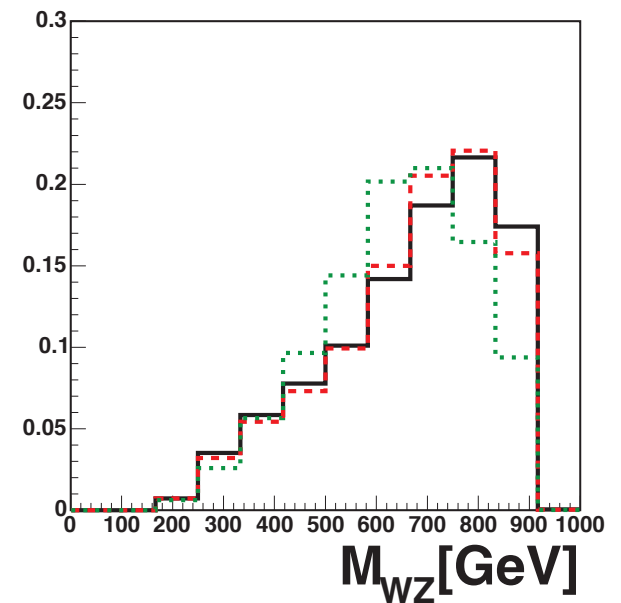
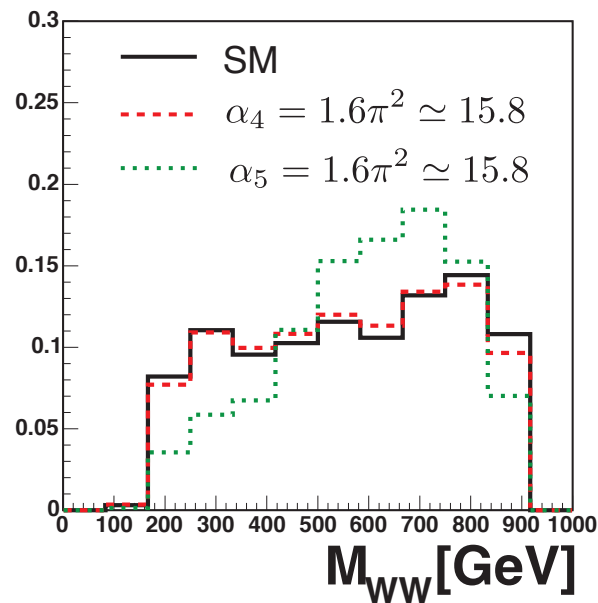
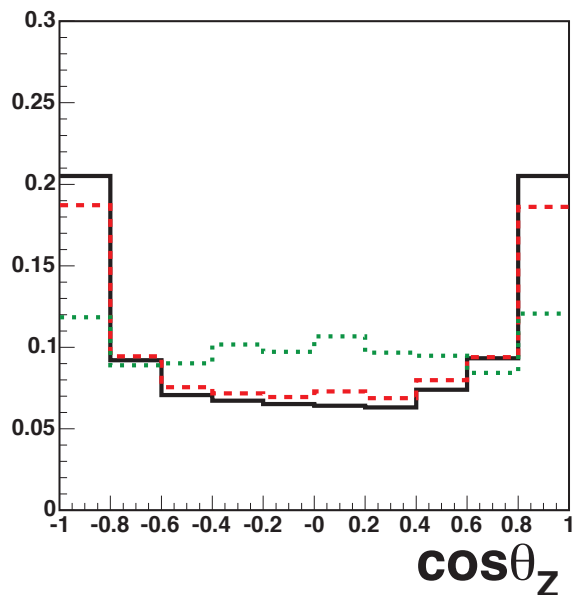
...for quartic gauge couplings

$$e^+e^- \rightarrow ZZZ$$

few SM backgrounds

$$\rightarrow WWZ$$

dominant background is  $t\bar{t}$   
(reduce using  $e_R^-$ )

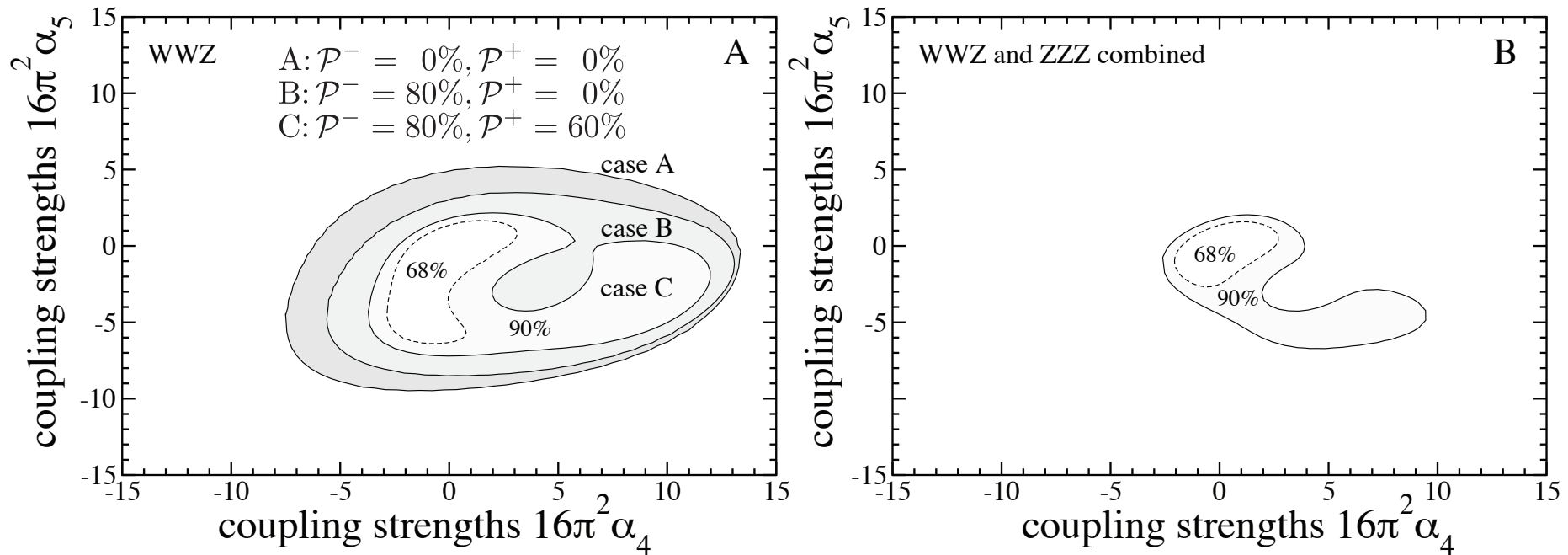


$$e^+e^- \rightarrow WWZ \quad 1 \text{ TeV}, \mathcal{P}^- = 80\%, \mathcal{P}^+ = 60\%, \mathcal{L} = 1000 \text{ fb}^{-1}$$

# Triple VB Production

...for quartic gauge couplings

- constraints on aQGC's:  $\alpha_4 \alpha_5$  in EW effective chiral Lagrangian  
(see backup slides)
- not great, factor  $\sim 30$  worse than LHC

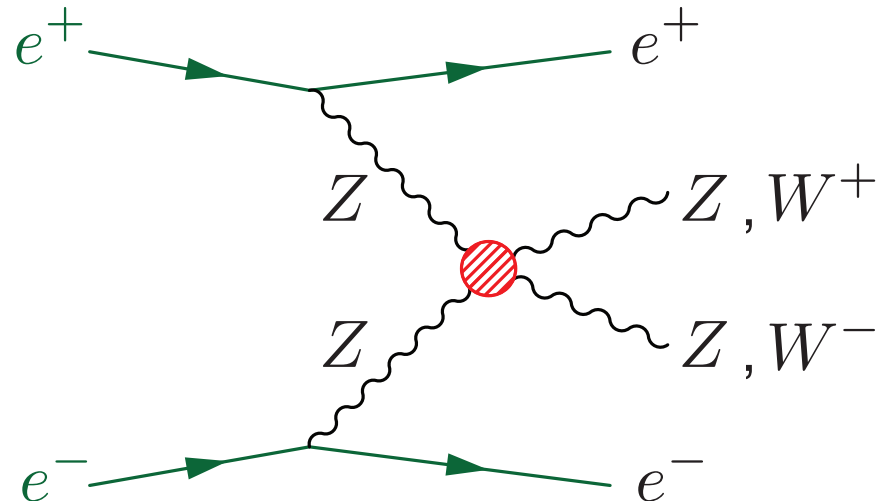
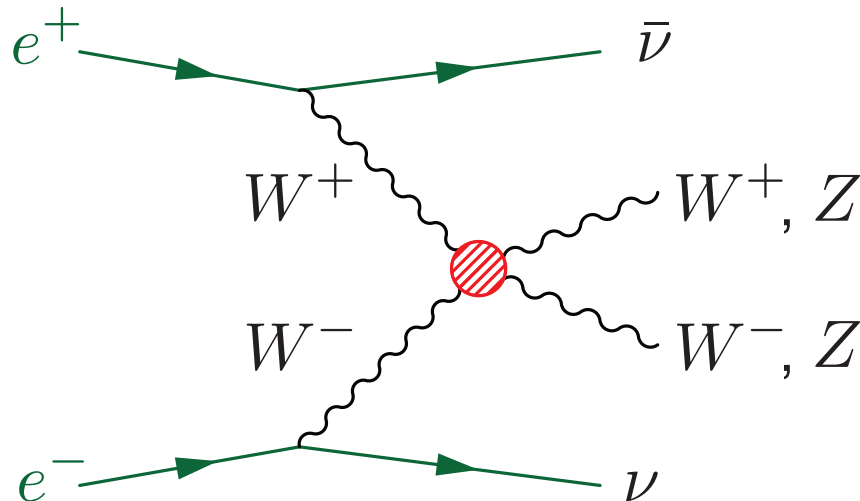


		WWZ			ZZZ	best
		no pol.	$e^-$ pol.	both pol.	no pol.	
$16\pi^2 \Delta\alpha_4$	$\sigma^+$	9.79	4.21	1.90	3.94	1.78
	$\sigma^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$16\pi^2 \Delta\alpha_5$	$\sigma^+$	3.05	2.69	1.17	3.94	1.14
	$\sigma^-$	-7.10	-6.40	-2.19	-3.53	-1.64

# Vector Boson Scattering

...for quartic gauge couplings

- any better?



- neutrinos instead of forward jets, **large missing invariant mass**
- in contrast to LHC, know the initial state in the scattering subprocess

- can scatter  $\gamma$ 's instead of  $Z$ 's

- c.f. 1 TeV ILC for subprocess energy to 14 TeV LHC

Falls ~short, higher energies better

$$p \rightarrow q$$

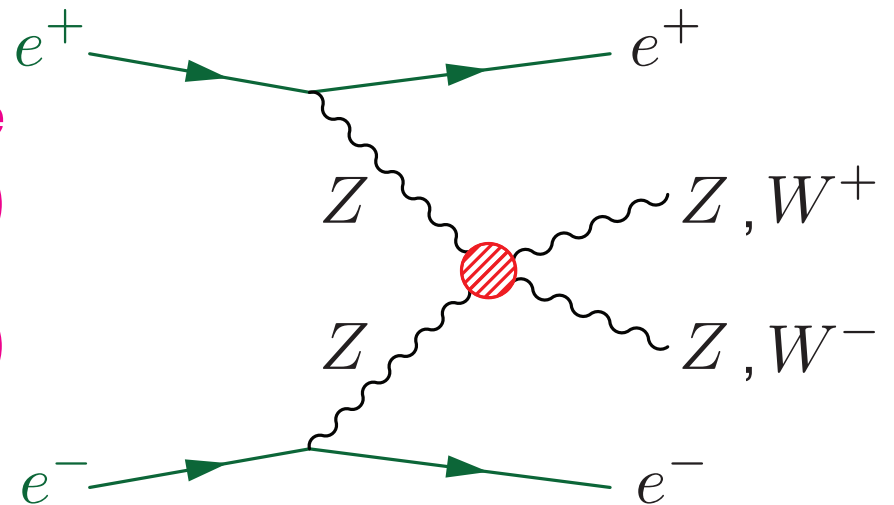
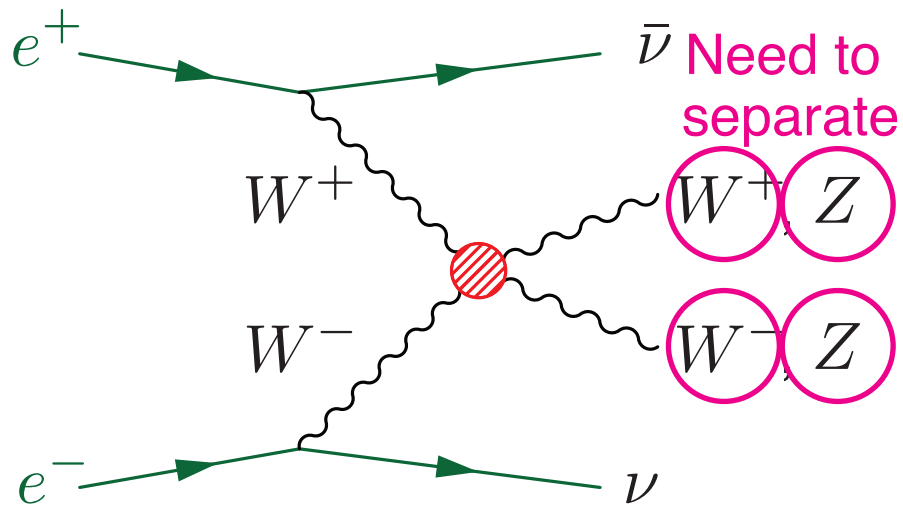
$$q \rightarrow W/Z$$

allows for an effective subprocess energy up to about 2 TeV

# Vector Boson Scattering

...for quartic gauge couplings

- any better?



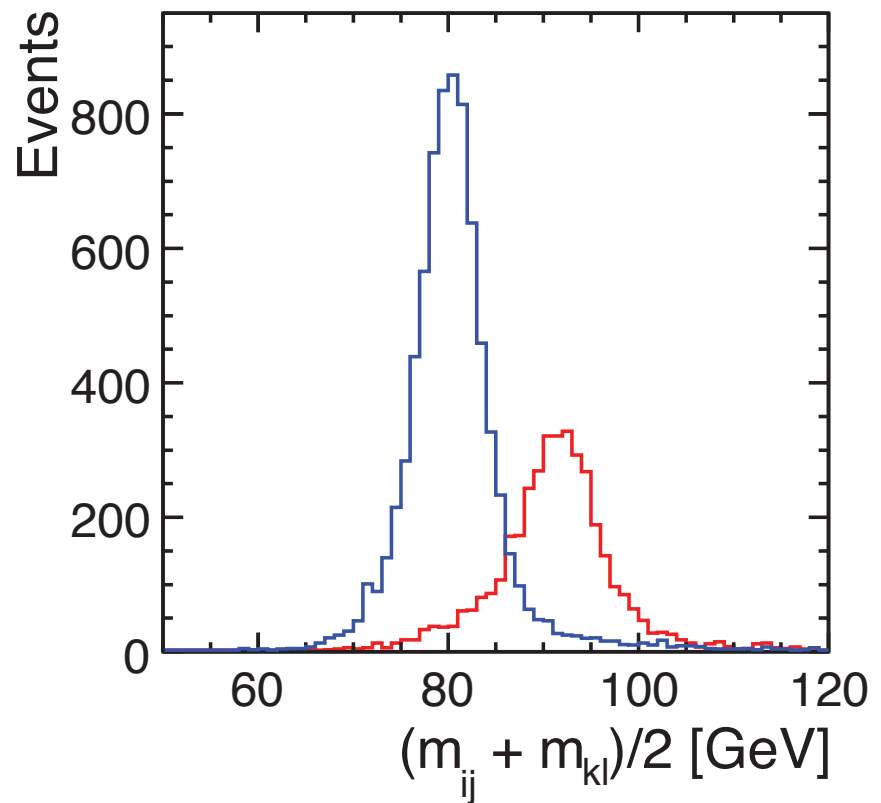
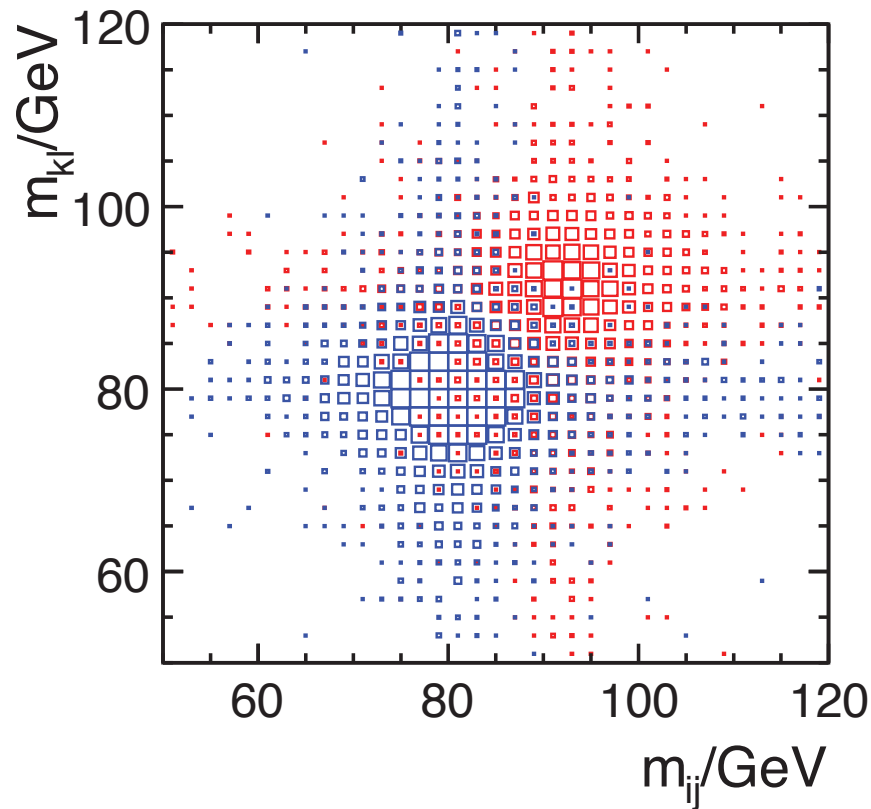
- polarization can change cross sections by up to factor 4

- can scatter  $\gamma$ 's instead of  $Z$ 's

# Vector Boson Scattering

...for quartic gauge couplings

- final state:  $e^+e^- \rightarrow \nu\bar{\nu}q\bar{q}q'\bar{q}'$  , “best” jet-jet combinations



$\nu_e\bar{\nu}_e WW$   $\nu_e\bar{\nu}_e ZZ$   
 $(W^+W^- \rightarrow W^+W^-)$   $(W^+W^- \rightarrow ZZ)$

Hadronic  $W/Z$  separation is  
 calorimeter benchmark  
 for ILC detectors  
 (this is the ILD detector)

# Vector Boson Scattering

...for quartic gauge couplings

Process	Subprocess	$\sigma$ [fb]
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q \bar{q} q \bar{q}$	$W^+W^- \rightarrow W^+W^-$	23.19
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q \bar{q} q \bar{q}$	$W^+W^- \rightarrow ZZ$	7.624
$e^+e^- \rightarrow \nu \bar{\nu} q \bar{q} q \bar{q}$	$V \rightarrow VVV$	9.344
$e^+e^- \rightarrow \nu e q \bar{q} q \bar{q}$	$WZ \rightarrow WZ$	132.3
$e^+e^- \rightarrow e^+e^- q \bar{q} q \bar{q}$	$ZZ \rightarrow ZZ$	2.09
$e^+e^- \rightarrow e^+e^- q \bar{q} q \bar{q}$	$ZZ \rightarrow W^+W^-$	414.
$e^+e^- \rightarrow b \bar{b} X$	$e^+e^- \rightarrow t \bar{t}$	331.768
$e^+e^- \rightarrow q \bar{q} q \bar{q}$	$e^+e^- \rightarrow W^+W^-$	3560.108
$e^+e^- \rightarrow q \bar{q} q \bar{q}$	$e^+e^- \rightarrow ZZ$	173.221
$e^+e^- \rightarrow e \nu q \bar{q}$	$e^+e^- \rightarrow e \nu W$	279.588
$e^+e^- \rightarrow e^+e^- q \bar{q}$	$e^+e^- \rightarrow e^+e^- Z$	134.935
$e^+e^- \rightarrow X$	$e^+e^- \rightarrow q \bar{q}$	1637.405

- typical cross sections for signals and backgrounds, 1 TeV

$$\mathcal{P}^- = 80\%, \mathcal{P}^+ = 40\% \quad 23$$

# Vector Boson Scattering

...for quartic gauge couplings

- sensitivities to EW effective chiral Lagrangian aQGC's

$e^+e^- \rightarrow$	$e^-e^- \rightarrow$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_{10}$
$W^+W^- \rightarrow W^+W^-$	$W^-W^- \rightarrow W^-W^-$	+	+	-	-	-
$W^+W^- \rightarrow ZZ$		+	+	+	+	-
$W^\pm Z \rightarrow W^\pm Z$	$W^-Z \rightarrow W^-Z$	+	+	+	+	-
$ZZ \rightarrow ZZ$	$ZZ \rightarrow ZZ$	+	+	+	+	+

Yes, could do  
electron-electron  
at LC

- e.g., ILD detector, full simulation,  
1 TeV,  $1000 \text{ fb}^{-1}$ ,  $\mathcal{P}^- = -80\%$ ,  $\mathcal{P}^+ = 30\%$

$$\begin{aligned} -1.38 < \alpha_4 < +1.10 \\ -0.92 < \alpha_5 < +0.77 \end{aligned} \quad (\text{at } 90\% \text{ CL})$$

arXiv:1006.3396 [hep-ex]

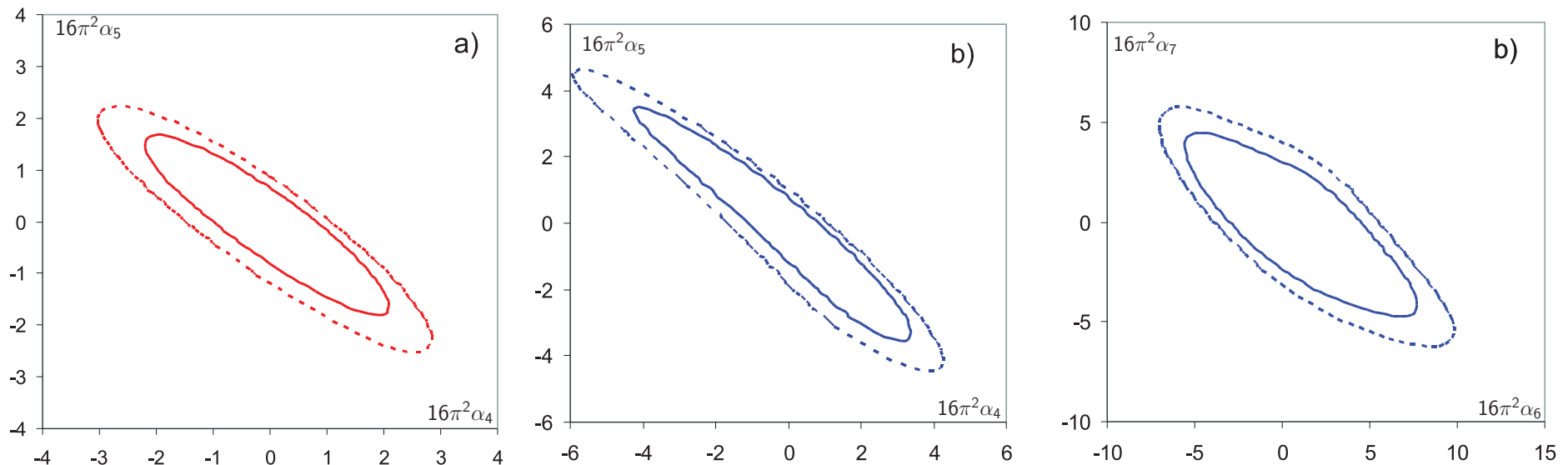


# Vector Boson Scattering

...for quartic gauge couplings

- similar TESLA analysis (fast simulation)

hep-ph/0604048



coupling	$\sigma-$	$\sigma+$
$\alpha_4$	-1.41	1.38
$\alpha_5$	-1.16	1.09

SU(2) custodial symmetry  
constrained

coupling	$\sigma-$	$\sigma+$
$\alpha_4$	-2.72	2.37
$\alpha_5$	-2.46	2.35
$\alpha_6$	-3.93	5.53
$\alpha_7$	-3.22	3.31
$\alpha_{10}$	-5.55	4.55

not  
constrained

## Vector Boson Scattering

...for quartic gauge couplings

- comparison to HL-LHC?

LC constraints significantly weaker by large factors:

parameter	300 fb <sup>-1</sup>	1 ab <sup>-1</sup>	3 ab <sup>-1</sup>
$\alpha_4$	0.066	0.025	0.016

ATLAS study, CERN preprint ATL-PHYS-PUB-2012-005, <http://cds.cern.ch/record/1496527>.

# Vector Boson Scattering

...for quartic gauge couplings

- Compare via limits on mass  $M$  of a broad resonance in simplified models obtained from limits on  $\alpha_4$  (larger limit better)

Type of resonance	LHC 300 fb <sup>-1</sup>		LHC 3000 fb <sup>-1</sup>	
	5 $\sigma$	95% CL	5 $\sigma$	95% CL
scalar $\phi$	1.8 TeV	2.0 TeV	2.2 TeV	3.3 TeV
vector $\rho$	2.3 TeV	2.6 TeV	2.9 TeV	4.4 TeV
tensor $f$	3.2 TeV	3.5 TeV	3.9 TeV	6.0 TeV

Best: derived from LHC  $W^\pm W^\pm$  channel with less background

Type of resonance	95% CL
scalar $\phi$	1.64 TeV
vector $\rho$	2.09 TeV
tensor $f$	2.76 TeV

ILC translated limits

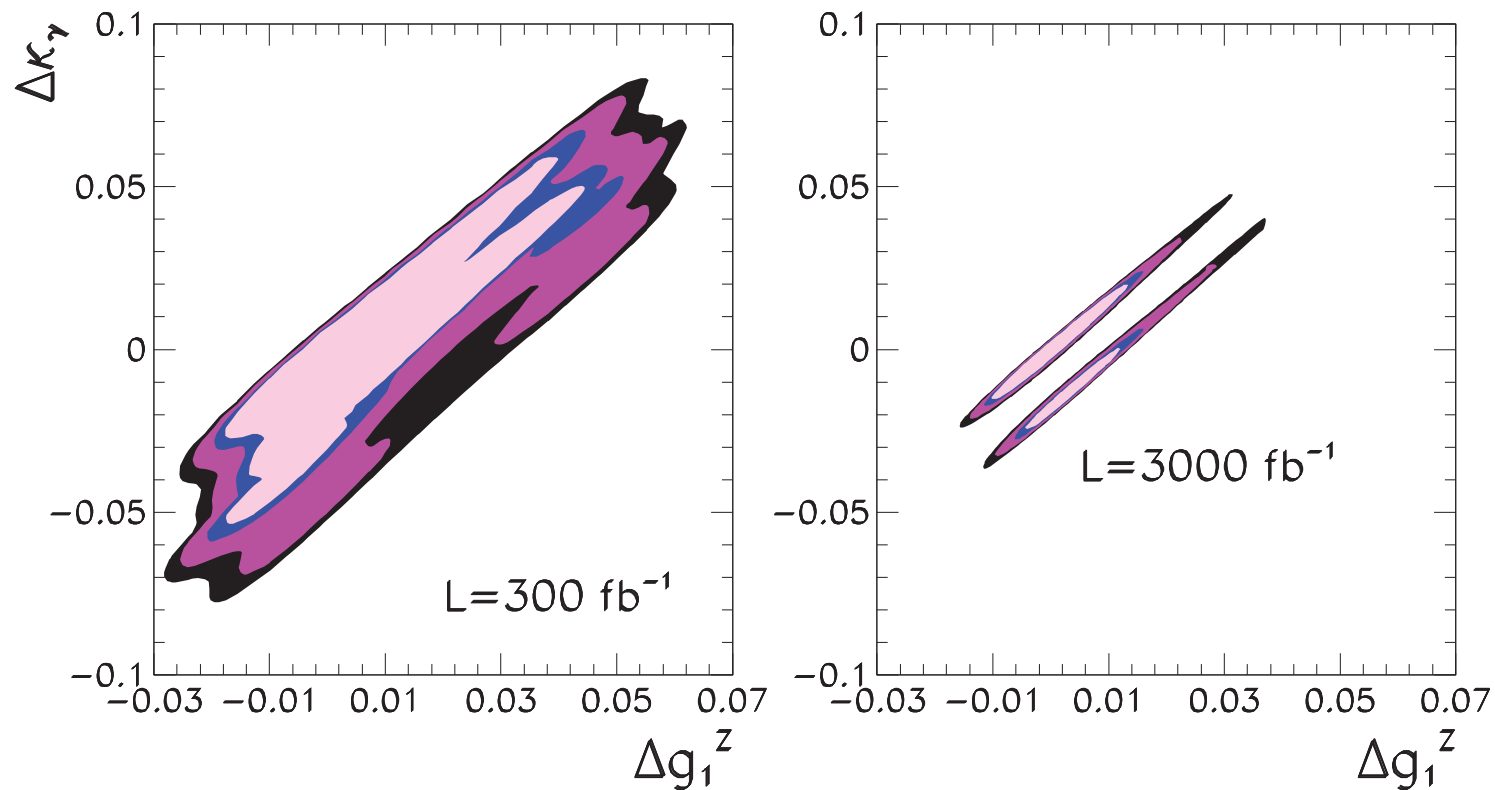
95% CL limits

Type of resonance	LHC 300 fb <sup>-1</sup>	LHC 3000 fb <sup>-1</sup>
scalar $\phi$	0.9 TeV	1.3 TeV
vector $\rho$	1.2 TeV	1.7 TeV
tensor $f$	1.6 TeV	2.3 TeV

Derived from LHC  $W^+ W^-$  channel with significant background

## TGC's from global fit to Higgs data

- Constraints due to analysis of projected Higgs properties data from LHC and HL-LHC, could then *combine* with direct measurements

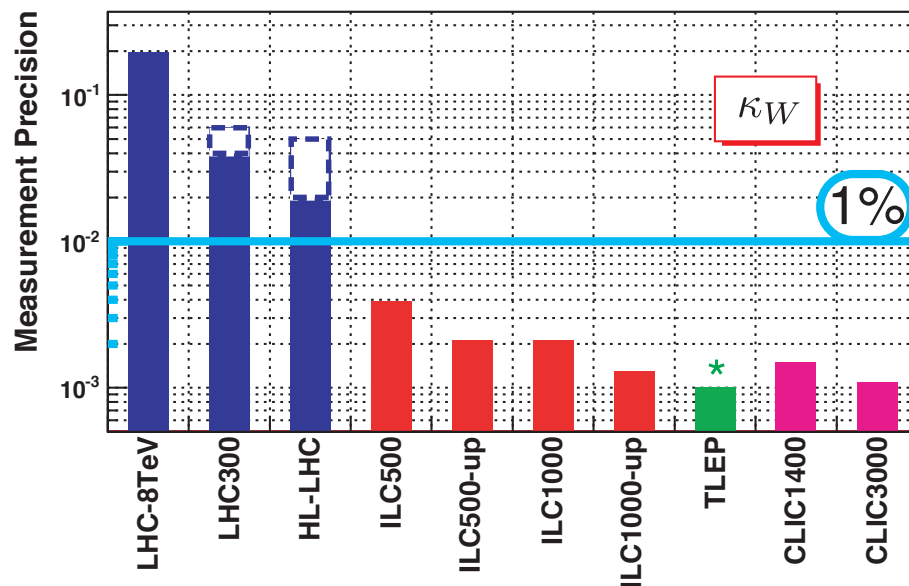


90%, 95%, 99%,  $3\sigma$  allowed regions

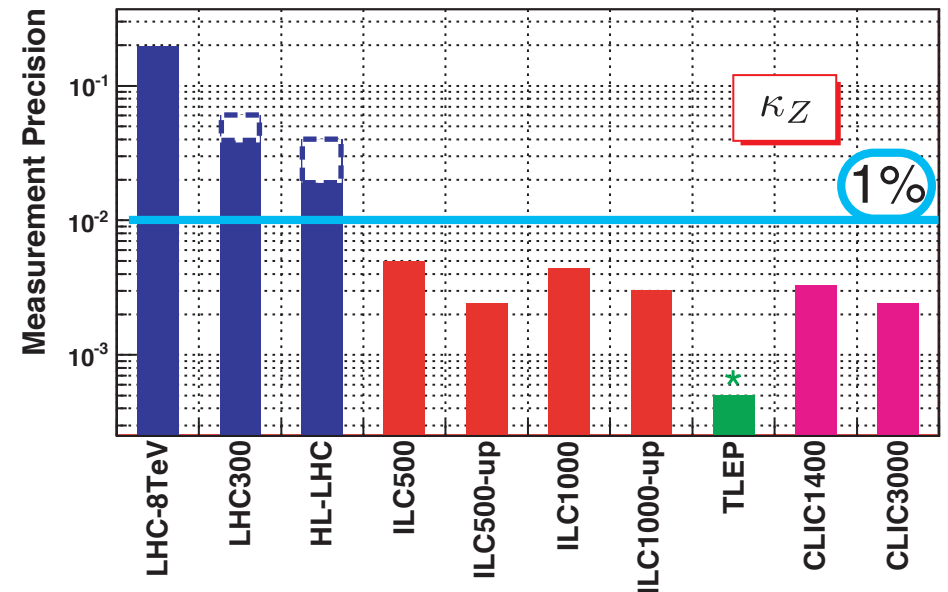
arXiv:1207.1344v3 [hep-ph] extended  
in Snowmass EW report

# TGC's from global fit to Higgs data

- What if analysis done using as input projected precisions on Higgs properties from linear collider options? Include aQGC's? e.g.:



Snowmass Higgs report, arXiv:1310.8361 [hep-ex]



- Precision on deviations of Higgs couplings:  $g_{HWW}$ ,  $g_{HZZ}$

Inquiries sent to authors – interested

## Summary/Conclusions

- LC options can bring significant new insights to multi-boson interactions
- Anomalous triple gauge couplings, induced by dim-6 operators, are significantly better probed by high-energy linear collider options
- Anomalous quartic gauge couplings, induced by dim-8 operators, are significantly better probed (by 1-2 orders of magnitude) by the LHC, due to the stronger growth of the anomalous cross section with energy
- Global fits with Higgs properties are useful
- Complementarities abound!

Verbatim from Snowmass  
EW report

## EW effective chiral Lagrangian

The deviations of the couplings from the SM values are expressed in terms of the  $\alpha_i$  parameters as

$$\Delta g_1^\gamma = 0 \qquad \Delta \kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8) \quad (4.5)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2}\alpha_3 \qquad \Delta \kappa^Z = \delta_Z - g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8) \quad (4.6)$$

and

$$\lambda^\gamma = -\frac{g^2}{2}(\alpha_1^\lambda + \alpha_2^\lambda) \qquad \lambda^Z = -\frac{g^2}{2}\left(\alpha_1^\lambda - \frac{s_w^2}{c_w^2}\alpha_2^\lambda\right) \quad (4.7)$$

Deviations from these SM values in the quartic couplings are introduced through the corrections induced by the  $\alpha_i$  to the couplings that preserve custodial  $SU(2)$  symmetry,

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \frac{g^p p}{c_w^2 - s_w^2}\alpha_1 + \frac{g^2}{c_w^2}\alpha_3 \quad (4.9a)$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}\alpha_4 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}\alpha_5 \quad (4.9b)$$

$$\Delta g_1^{WW} = 2c_w^2\Delta g_1^{\gamma Z} + g^2\alpha_4 \qquad \Delta g_2^{WW} = 2c_w^2\Delta g_1^{\gamma Z} - g^2(\alpha_4 + 2\alpha_5) \quad (4.9c)$$

$$h^{ZZ} = g^2(\alpha_4 + \alpha_5). \quad (4.9d)$$

## EW effective chiral Lagrangian

Propagator/oblique  $\mathcal{L}'_0 = \frac{v^2}{4} \text{tr} \{ \mathbf{T} \mathbf{V}_\mu \} \text{tr} \{ \mathbf{T} \mathbf{V}^\mu \}$

Propagator/oblique  $\mathcal{L}_1 = gg' \text{tr} \{ \mathbf{B}_{\mu\nu} \mathbf{W}^{\mu\nu} \}$

aTGC  $\mathcal{L}_2 = ig' \text{tr} \{ \mathbf{B}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu] \}$

aTGC  $\mathcal{L}_3 = ig \text{tr} \{ \mathbf{W}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu] \}$

aQGC  $\mathcal{L}_4 = (\text{tr} \{ \mathbf{V}_\mu \mathbf{V}_\nu \})^2$

aQGC  $\mathcal{L}_5 = (\text{tr} \{ \mathbf{V}_\mu \mathbf{V}^\mu \})^2$

aQGC  $\mathcal{L}_6 = \text{tr} \{ \mathbf{V}_\mu \mathbf{V}_\nu \} \text{tr} \{ \mathbf{T} \mathbf{V}^\mu \} \text{tr} \{ \mathbf{T} \mathbf{V}^\nu \}$

aQGC  $\mathcal{L}_7 = \text{tr} \{ \mathbf{V}_\mu \mathbf{V}^\mu \} (\text{tr} \{ \mathbf{T} \mathbf{V}_\nu \})^2$

Propagator/oblique  $\mathcal{L}_8 = \frac{1}{4} g^2 (\text{tr} \{ \mathbf{T} \mathbf{W}_{\mu\nu} \})^2$

aTGC  $\mathcal{L}_9 = \frac{1}{2} ig \text{tr} \{ \mathbf{T} \mathbf{W}_{\mu\nu} \} \text{tr} \{ \mathbf{T} [\mathbf{V}^\mu, \mathbf{V}^\nu] \}$

aQGC  $\mathcal{L}_{10} = \frac{1}{2} (\text{tr} \{ \mathbf{T} \mathbf{V}_\mu \})^2 (\text{tr} \{ \mathbf{T} \mathbf{V}_\nu \})^2$